U.S. DEPARTMENT OF COMMERCE
National Technical Information Service

AD-A024 190

DECISION AND CONTROL MODELS IN OPERATIONS RESEARCH

YALE UNIVERSITY

PREPARED FOR
ARMY RESEARCH OFFICE

27 FEBRUARY 1976

134146

ARO 11639.5-M

AD A024190

Decision and Control Models in Operations Research

Final Report

Harvey M. Wagner

February 27, 1976

U. S. Army Research Office

DAHC04 74 G 0088

DAHC04 75 G 0079

Yale University

Approved for Public Release;
Distribution Unlimited.

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

22

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: F 1	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Decision and Control Models in Operations Research	5. TYPE OF REPORT & PERIOD COVERED: Final 1 Jan. 1974 - 31 Dec. 1975
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Harvey M. Wagner	8. CONTRACT OR GRANT NUMBER(s): DAHC04 74 G 0088, DAHC04 75 G 0079
9. PERFORMING ORGANIZATION NAME AND ADDRESS: School of Organization and Management, Yale University, 56 Hillhouse Avenue, New Haven, Connecticut 06520	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: U. S. Army Research Office, Post Office Box 12211, Research Triangle Park, NC 27709	12. REPORT DATE: February 27, 1976
	13. NUMBER OF PAGES: 22
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inventory replenishment; (s,S) policies; computer simulation; variance reduction; statistical estimation of demand; inventory systems design.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research concentrated on one major topic of investigation: statistical properties of inventory models.

The analysis dealt with comparing the operating characteristics of several important stockage rules when the underlying demand distribution parameters were estimated from a limited sample of historical data (say, 13, 26, or 52 observations). Particular attention was given to the frequently

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block #20 continued.

observed case where the probability of zero demand occurring within any time period was significantly large (hence, say, half of the historical observations indicated zero demand), but the positive levels of demand were not necessarily small. The context of the analyses was a large-scale inventory system containing thousands of individual items, and thus, the computational burden of implementing the stockage policies was recognized in the comparison of different stockage rules. The research approach encompassed applied Markovian probability analysis and computer simulation analysis containing sophisticated statistical autoregressive time series methods.

ABSTRACT

The research concentrated on one major topic of investigation: statistical properties of inventory models.

The analysis dealt with comparing the operating characteristics of several important stockage rules when the underlying demand distribution parameters were estimated from a limited sample of historical data (say, 13, 26, or 52 observations). Particular attention was given to the frequently observed case where the probability of zero demand occurring within any time period was significantly large (hence, say, half of the historical observations indicated zero demand), but the positive levels of demand were not necessarily small. The context of the analyses was a large-scale inventory system containing thousands of individual items, and thus, the computational burden of implementing the stockage policies was recognized in the comparison of different stockage rules. The research approach encompassed applied Markovian probability analysis and computer simulation analysis containing sophisticated statistical autoregressive time series methods.

STATISTICAL PROPERTIES OF INVENTORY MODELS

Review of Work

Over the past two decades, mathematical analysis of inventory stockage models has made great progress, and real-life implementation of inventory systems based, at least in part, on the results of this modern research has taken place. But typically practitioners have applied theoretically derived formulas to actual situations in which the model's underlying assumptions are far from being met. A critical divergence between the model's assumptions and reality occurs with regard to the amount of knowledge the systems' designer has about the underlying demand distributions. With rare exceptions, theoretical inventory models assume that the probability distribution of an item's demand is completely specifiable. In the most sophisticated theoretical inventory models, heavy reliance is placed on this assumption in deriving an algorithm for computing an optimal policy. Many other theoretical analyses make gross assumptions that permit stating inventory formulas which require only a few parameters of the demand distribution; but even then, these approximate analyses assume that the small number of parameters are known with certainty. A practitioner utilizing such models typically employs standard statistical procedures to estimate the parameters in the approximation formulas, and pays little if any attention to modifying the estimates due to the special nature of the application. There has been a significant knowledge gap concerning the nature of the systematic performance and prediction biases that are introduced when these statistical estimates are substituted for parameters in inventory models.

The seriousness of the problem stems in part from the fact that in most real situations, only a very limited amount of past data on demand are available, and frequently such data contain a large proportion of zero-demand observations. For example, in a 26 week history of demand for an item, perhaps no demand occurred during 10 of the weeks, and in the remaining 16 weeks, the demand values displayed wide variation (such as, 30, 100, 7, 245, etc.). We currently have no scientific understanding of what stockage rules perform acceptably well in such an environment, and how much accuracy we have in our estimates of future system performance based on the past data. The following scenario will clarify further the context for the work supported.

Consider an inventory manager who must design a system of replenishment rules for the stockage of possibly thousands of items. Assume that, without too much difficulty the manager can specify the parameters of the criterion function, so that the manager has a direct way of determining whether one system is better than another, provided that the manager can obtain the relevant associated operating characteristics for each system. For example, assume that the manager knows the costs of holding inventory, making replenishment decisions, reviewing the stock status of an item, etc. Also assume that the manager is able to articulate objectives or costs relating to stockouts; to illustrate, the manager may specify a cost attached to a unit of demand that cannot be filled, or perhaps the manager may set a limit on the probability of a stockout, or possibly the manager may target fulfilling a certain fraction of total demand.

In designing the system, the manager must select a class of decision rules, and preferably, pick optimal policies from the selected class. To illustrate, the complete economic preference function of the manager may imply that if the manager knows the probability distribution of demand, then an optimal policy is of an (s,S) form: when inventory on hand plus on order falls below s, order so that, as a consequence, inventory on hand plus on order equals S. The research of Veinott and Wagner shows how to compute optimal values for (s,S) in this circumstance, and the research of Wagner, O'Hagen, and Lundh tests out various numerical approximations that, since the publication of that paper, have turned out to be implementable in practice. But if the manager faces considerable uncertainty about each item's *demand distribution*, because the manager has only a limited number of past observations on demand, it is by no means obvious as to how the manager should proceed. In fact, the decision process for systems design becomes much more complex than the mere selection of parameter values for (s,S).

First, the manager must decide how much past demand data to actually use. If the manager suspects that the underlying forces causing the demand for an item shift from time to time the manager may want to ignore data that are older than some number of periods, which the manager then must specify. Since the system is to operate over an indefinite future, the manager also must determine how often to update the rules, that is, how often to discard old historical data and recompute the replenishment policies' parameters. For example, the manager may choose to revise the policies every six months and, at each revision, utilize the past twelve months' data.

Second, since the manager does not know the form of the demand distributions, the manager must either guess at the form, then statistically estimate the distribution's parameters for each item (possibly using a prior distribution), and finally compute policies assuming that the estimated distributions are in fact the true distributions; or the manager may choose to use an "approximately optimal" form for the replenishment *policy* (such as let S equal a number of mean demands plus a multiple of the standard deviation of demand, and $s=S-D$, where D is the familiar Wilson square root lot size formula that requires knowing only the mean demand), and then select statistical estimators for the parameters required by this policy form. This enumeration does not make apparent the full design decision possibilities because, within each option, the manager has many alternatives from which to choose. To complete the scenario, suppose that the manager does make a particular selection from among all these many options.

Third, the manager must decide *before* implementing the system whether the design parameters are well set. For example, if the manager lets the stockage objective level $S = a\cdot$(mean demand) $+ b\cdot$(standard deviation of demand), the manager must decide on the appropriate values for a and b. Since the manager realizes that there may have been a compounding approximation on approximation, the manager usually hesitates to rely solely on theoretical probability distribution analysis that is based on complete knowledge of the demand distribution. Most likely, the manager will employ retrospective simulation (that is, will reuse the limited amount of past data from which the manager estimated the mean demand and the standard deviation of demand) to adjust the values of a and b so that the resultant *statistical estimate* of the criterion function is at an optimal value. (Actually, the manager probabily will set the same values for a and b over a large number of items, and so will perform the adjustment process via retrospective simulation by using an aggregate objective function.) The manager also will estimate several operating characteristics of importance, such as average inventory on hand, the probability of a stockout, the fraction of demand filled, etc.

To summarize the scenario, we see that the system's designer must select in concert the number of historical observations to use, the frequencey for repeating the reestimation process, the form of the replenishment rule, the statistical estimators to produce the demand parameters required by the rule, and the design parameters of the rule. The manager makes all of these choices based, at least in part, on simulating how the proposed system would have performed in the past (and in doing so, the manager typically uses the same limited data for both estimation and prediction). In all the discussion to follow, we employ the term *system's design* to mean the entire composite of these many choices.

The nature of the supported research was to examine:

1. How good are the manager's statistical estimates of the system's future performance for selected choices of the system's design? Similarly, how good are theoretically based estimates of future performance?

2. Under what circumstances do certain "approximately optimal" policies perform better than others? Are some policies statistically more robust than others?

3. How do the statistical estimates of the system's future performance, as well as the performance characteristics themselves, depend on choice parameters such as the number of historical observations to use and the frequency of reestimating the policy's demand parameters?

Special attention was given to systems in which the demand distributions displayed a high frequency of zero demand (the positive levels of demand, however, did not need to be anywhere near zero).

Except under extremely special assumptions, it was not possible to derive computationally practical formulas for the exact statistical distributions associated with the above questions. The best hope for increasing our ability to answer the research questions above in a wide variety of situations was to derive more workable numerical approximations. With that objective in mind, we built under the research grants a computer simulation model that yielded the actual operating characteristics and the corresponding properties of statistical estimators of these operating characteristics for inventory systems that contained uncertainty about the demand distributions. The details of the simulation design are discussed below.

Exploratory Simulation Experiments. The selection of the underlying probability distributions and other parameters used in the research was influenced by the earlier study of Wagner, O'Hagen, and Lundh. The demand distributions were both Poisson and Negative Binomial (variance/mean = 3, 9), as well as distributions that were probability mixtures of zero demand and a standard Negative Binomial distribution (variance/mean = 9). More than 144 sets of the economic and lead-time parameters were investigated. A summary of the case selections is shown in Table 1; unit holding cost was normalized to equal 1. A few additional cases with higher mean demands also were run.

Table 1

Demand Distribution	Mean Demand	Unit Penalty Cost	Setup Cost	Lead Time
Poisson	2,4,8,16	4,9,99	8,32,64	0,2,4
Negative Binomial (V/μ = 3, 9)	2,4,8,16	4,9,99	8,32,64	0,2,4

Research concentrated on investigating optimal and approximately optimal (s,S) policies assuming that demand for each period is independently and identically distributed. For statistically derived policies, the formulas in Wagner, O'Hagen, and Lundh were used, where every J periods the demand history over the past K periods was employed to update the estimates of the mean and variance of the demand distribution. The values for (J,K) that have been tested are (13,13), (13,26), and (26,26). Each time the policies were revised, forecasts of important operating characteristics were made using restrospective simulation over the past L periods. The values for (J, K, L) that were tested are (13,13,13), (13,26,13), (13,26,26), and (26,26,26).

Summary of Results

An illustration of the simulation model output is shown in Figure 1. The case was a Negative Binomial distribution for demand with a mean of 4 per period, a lead time of 4 periods, a unit penalty cost of 99, a setup cost of 64, and a unit holding cost of 1. An optimal policy was (33,57) under these assumptions, with expected cost per period equal to 40.50. [This policy was obtained in the Wagner, O'Hagen, and Lundh study.] The simulations in Figs. 1a and 1b were run for 200 cycles, each containing 13 periods; the simulations in Fig. 1d were run for 200 cycles, each containing 26 periods. Simulation estimates, and their standard errors, are shown for period-end inventory level, stockout quantity, stockout frequency, replenishment quantity, replenishment frequency, and total cost per period. The Normal approximation policy (32,55) also was simulated--in this particular illustration, it outperformed the optimal policy, because of the particular realized history of random demands. The bottom half of Figs. 1a, 1b, and 1d show the behavior of the statistically based policies and the corresponding forecasts. In the (13,13) simulation,

the average cost per period for the statistical policy was 51.52 (standard error 3.549), and the forecasted average cost per period using 13 periods for the retrospective simulations was 34.66 (standard error .618).

An illustrative tabulation of resulting total cost per period under optimal (s,S) policies, approximately optimal policies, and statistical policies, is shown in Fig. 2 for all cases of Negative Binomial demand with lead time 4 . Similar tabulations are available for all the Poisson cases and the Negative Binomial cases with other lead times. Similar tabulations also are available for other operating characteristics, including standard deviation of total cost per period, period-end inventory level, backlog as a proportion of mean demand, frequency of backlog, and replenishment frequency.

An illustrative tabulation of the forecast biases is given in Fig. 3 for all the cases of Negative Ninomial demand with lead time 4 . Average values for the statistically determined (s,S) policy parameters were tabulated, as illustrated in Fig. 4. These averages were remarkably close to the optimal values for (s,S).

Because of the considerable magnitude in many of the standard errors of the estimated mean values for the operating characteristics, the above tabulations provided "at a glance" only orders of magnitude and plausible relationships, but still left open to question the regularity of certain observed behavior. Hence, another means of summarization was used to clarify the results.

The cases shown in Table 1 above were aggregated into multi-item inventory systems, namely, the 72 Poisson demand cases and the two sets of 72 Negative Binomial cases. A large number of such summarizations for the various operating characteristics were tabulated, and a few will be illustrated below to show their format and suggest their implications.

Figures 5a and 5c, for Poisson and Negative Binomial (variance/mean = 3) demand, respectively, tabulated the proportion of total cost associated with the various parameter values in each 72 item system that employed statistical policies which were recomputed each 26 periods using 26 periods data. For example, in the Poisson system, the total system's cost per period was 2037.8, of which 39.9% was due to items with penalty cost 99, 56.6% to items with setup cost 64, 38.1% to items with mean demand 16, 15% to items with penalty cost 99 and mean demand 16, etc. Figures 5b and 5d, for Poisson and Negative Binomial demand, respectively, tabulate the percentages by which the statistical system's total cost per period exceeded that of the optimal policy. For example, in the Negative Binomial system, total system cost for statistical policies exceeded that of optimal policies by only 5.8%.

Similar multi-item systems summaries were tabulated for the important operating characteristics of interest, including period-end inventory, backlog cost, backlog frequency, replenishment cost, and replenishment frequency.

The forecasts were compared in a similar manner. Sources of the underestimation bias for a statistical system that was revised every 26 weeks using 26 weeks of past data to set the stockage parameters and retrospectively simulate is shown in Fig. 6 for the Negative Binomial items. In this illustrative example, total cost per period was underestimated by 9.3% for the total system comprised of 72 items. Similar summaries were tabulated for other settings of the review frequency and for other operating characteristics.

The following reports summarize the results and implications of all the described simulation experiments:

STATISTICAL PROBLEMS IN INVENTORY CONTROL, Alastair MacCormick, December 1974, 244 pp.

MULTI-ITEM INVENTORY SYSTEM POLICIES USING STATISTICAL ESTIMATES: NEGATIVE BINOMIAL DEMANDS, Arthur S. Estey and Ronald L. Kaufman, September 1975, 85 pp.

VARIANCE REDUCTION TECHNIQUES FOR AN INVENTORY SIMULATION, Richard Ehrhardt, September 1975, 24 pp.

COMPUTER PROGRAMS FOR (s,S) POLICIES UNDER INDEPENDENT OR FILTERED DEMANDS, Ronald L. Kaufman, (in preparation), 71 pp.

MULTI-ITEM INVENTORY SYSTEM POLICIES USING STATISTICAL ESTIMATES: SPORADIC DEMANDS, Ronald L. Kaufman and John G. Klincewicz, (in preparation), 83 pp.

THE POWER APPROXIMATION, A NEW ALGORITHM FOR CONTROLLING INVENTORY SYSTEMS WITH INCOMPLETE DEMAND INFORMATION, Richard Ehrhardt, (in preparation), 102 pp.

7.

STATISTICAL PROPERTIES OF INVENTORY SYSTEMS

EXPERIMENT # : 5452
TIME/DATE : 1736/072071

SYSTEM ASSUMPTIONS:

*QUANTITY DEMANDED HAS NEGATIVE BINOMIAL DISTRIBUTION, MEAN = 4, VARIANCE/MEAN RATIO = 3
*STOCKOUTS BACKLOGGED, REPLENISHMENT LEAD TIME = 4
*(s,S) REPLENISHMENT POLICY
*COST RATIOS: C(OUT)/C(IP) = 99, C(FIX)/C(IP) = 64

SAMPLE STATISTICS OF SYSTEM OPERATING CHARACTERISTICS, SAMPLE SIZE 200 CYCLES OF 13 PERIODS:

OPERATING CHARACTERISTIC	SAMPLE MEAN OF O.C.	S.D. OF MEAN	SAMPLE S.D. OF O.C.	RESIDUAL S.D.	AUTOREGRESSIVE ORDER	CONFIDENCE POINTS 0.025L	0.025U
*DECISION RULE TO SET (s,S): DYNAMIC PROGRAM (OPTIMAL) (s,S) = (57, 13), EXPECTED COST = 40.50							
MEAN OVER 13 PERIODS OF:							
PERIOD-END INVENTORY	27.40	0.310	4.5	4.5	0	26.78	28.03
STOCKOUT QUANTITY	[illegible]	0.0157	0.22	0.22	0	0.006	0.068
STOCKOUT FREQUENCY	0.006	0.0017	0.024	0.024	0	0.002	0.009
REPLENISHMENT QUANTITY	[illegible]	[illegible]	1.25	1.25	0	3.74	4.08
REPLENISHMENT FREQUENCY	0.145	0.0032	0.045	0.045	0	0.139	0.152
COST PER PERIOD	40.40	1.499	21.2	21.2	0	37.45	43.36
*DECISION RULE TO SET (s,S): MORSE UNIT NORMAL APPROXIMATION (s,S) = (55, 12), EXPECTED COST = 40.63							
MEAN OVER 13 PERIODS OF:							
PERIOD-END INVENTORY	[illegible]	[illegible]	4.5	4.5	0	25.33	26.59
STOCKOUT QUANTITY	[illegible]	0.0121	0.17	0.17	0	0.010	0.058
STOCKOUT FREQUENCY	0.007	0.0020	0.028	0.028	0	0.003	0.011
REPLENISHMENT QUANTITY	[illegible]	[illegible]	1.22	1.22	0	3.74	[illegible]
REPLENISHMENT FREQUENCY	[illegible]	0.0032	0.046	0.046	0	0.145	0.158
COST PER PERIOD	[illegible]	1.167	16.5	16.5	0	36.69	41.29
*DECISION RULE TO SET (s,S): STATISTICAL NORMAL APPROXIMATION (PARAMETERS REVISED EVERY 13 PERIODS USING 13 PERIODS DATA)							
BIG S	[illegible]	0.75	10.6	10.6	0	51.3	54.2
LITTLE S	30.5	0.57	8.0	8.0	0	29.4	31.6
MEAN OVER 13 PERIODS OF:							
PERIOD-END INVENTORY	25.22	0.360	10.1	[illegible]	3	24.51	25.94
STOCKOUT QUANTITY	0.17	[illegible]	0.54	0.54	0	0.095	0.245
STOCKOUT FREQUENCY	0.032	0.0056	0.080	0.080	0	0.021	0.043
REPLENISHMENT QUANTITY	[illegible]	[illegible]	1.79	1.54	2	3.75	4.06
REPLENISHMENT FREQUENCY	0.147	[illegible]	0.056	0.050	2	0.143	0.152
COST PER PERIOD	51.52	[illegible]	50.2	50.2	0	44.52	58.52
FORECASTS:							
FROM 13 PERIODS' HISTORY:							
PERIOD-END INVENTORY	[illegible]	0.430	6.1	6.1	0	24.08	25.77
STOCKOUT QUANTITY	[illegible]	0.0031	0.04	0.04	0	-0.002	0.010
STOCKOUT FREQUENCY	0.002	0.0009	0.013	0.013	0	-0.000	0.003
REPLENISHMENT QUANTITY	[illegible]	[illegible]	[illegible]	1.64	1	3.71	4.09
REPLENISHMENT FREQUENCY	0.146	0.0034	0.049	0.049	0	0.139	0.153
COST PER PERIOD	[illegible]	[illegible]	8.7	8.7	0	33.44	35.88
ACTUAL VALUES LESS FORECASTS: (13-PERIOD FORECASTS)							
PERIOD-END INVENTORY	0.30	0.231	6.7	6.1	3	-0.15	0.76
STOCKOUT QUANTITY	0.17	0.0383	0.54	0.54	0	0.091	0.242
STOCKOUT FREQUENCY	0.030	0.0058	0.081	0.081	0	0.019	0.042
REPLENISHMENT QUANTITY	0.00	0.106	1.51	1.51	0	-0.21	0.21
REPLENISHMENT FREQUENCY	0.001	0.0042	0.059	0.059	0	-0.007	0.009
COST PER PERIOD	16.66	3.724	52.7	52.7	0	9.52	24.21

Figure 1a

STATISTICAL PROPERTIES OF INVENTORY SYSTEMS

EXPERIMENT # : 5453
TIME/DATE : 1737/072073

SYSTEM ASSUMPTIONS:

- QUANTITY DEMANDED HAS NEGATIVE BINOMIAL DISTRIBUTION, MEAN = 4, VARIANCE/MEAN RATIO = 3
- STOCKOUTS BACKLOGGED, REPLENISHMENT LEAD TIME = 4
- S,S REPLENISHMENT POLICY
- COST RATIOS: C(OUT)/C(IN) = 99, C(FIX)/C(IN) = 64

SAMPLE STATISTICS OF SYSTEM OPERATING CHARACTERISTICS, SAMPLE SIZE 200 CYCLES OF 13 PERIODS:

OPERATING CHARACTERISTIC	SAMPLE MEAN OF O.C.	S.D. OF MEAN	SAMPLE S.D. OF O.C.	RESIDUAL S.D.	AUTOREGRESSIVE ORDER	CONFIDENCE POINTS 0.025L	0.025U
•DECISION RULE TO SET (S,S): DYNAMIC PROGRAM (OPTIMAL) (S,S) = (57,[illegible]), EXPECTED COST = 40.50							
MEAN OVER 13 PERIODS OF:							
PERIOD-END INVENTORY	26.93	0.296	4.2	4.2	0	26.35	27.51
STOCKOUT QUANTITY	0.03	0.0121	0.17	0.17	0	0.006	0.054
STOCKOUT FREQUENCY	0.007	0.0018	0.025	0.025	0	0.003	0.010
REPLENISHMENT QUANTITY	3.99	0.083	1.17	1.17	0	3.83	4.15
REPLENISHMENT FREQUENCY	0.149	0.0025	0.044	0.043	1	0.144	0.154
COST PER PERIOD	39.41	1.183	16.7	16.7	0	37.08	41.75
•DECISION RULE TO SET (S,S): MORSE UNIT NORMAL APPROXIMATION (S,S) = (55,[illegible]), EXPECTED COST = 40.63							
MEAN OVER 13 PERIODS OF:							
PERIOD-END INVENTORY	25.44	0.292	4.1	4.1	0	24.87	26.02
STOCKOUT QUANTITY	0.03	0.0105	0.15	0.15	0	0.011	0.055
STOCKOUT FREQUENCY	0.010	0.0025	0.036	0.036	0	0.005	0.015
REPLENISHMENT QUANTITY	3.99	0.081	1.16	1.11	1	3.87	4.11
REPLENISHMENT FREQUENCY	0.155	0.0023	0.045	0.043	1	0.150	0.159
COST PER PERIOD	38.46	1.008	14.2	14.2	0	36.38	40.45
•DECISION RULE TO SET (S,S): STATISTICAL NORMAL APPROXIMATION (PARAMETERS REVISED EVERY 13 PERIODS USING 26 PERIODS DATA)							
BIG S	53.7	0.67	6.9	5.2	6	52.4	55.1
LITTLE S	31.2	0.52	5.1	3.9	6	30.2	32.2
MEAN OVER 13 PERIODS OF:							
PERIOD-END INVENTORY	24.68	0.500	7.1	7.1	0	23.70	25.67
STOCKOUT QUANTITY	0.08	0.0215	0.30	0.30	0	0.038	0.123
STOCKOUT FREQUENCY	0.017	0.0033	0.047	0.047	0	0.010	0.023
REPLENISHMENT QUANTITY	3.99	0.086	1.22	1.22	0	3.82	4.16
REPLENISHMENT FREQUENCY	0.156	0.0020	0.046	0.044	3	0.152	0.160
COST PER PERIOD	42.61	2.012	28.5	28.5	0	38.64	46.58
FORECASTS:							
FROM 13 PERIODS' HISTORY:							
PERIOD-END INVENTORY	24.59	0.318	4.5	4.5	0	23.96	25.22
STOCKOUT QUANTITY	0.01	0.0050	0.07	0.07	0	0.004	0.024
STOCKOUT FREQUENCY	0.005	0.0014	0.020	0.020	0	0.002	0.009
REPLENISHMENT QUANTITY	4.06	0.106	1.50	1.50	0	3.85	4.26
REPLENISHMENT FREQUENCY	0.156	0.0024	0.050	0.048	2	0.151	0.161
COST PER PERIOD	35.99	0.637	9.0	9.0	0	34.74	37.25
FROM 26 PERIODS' HISTORY:							
PERIOD-END INVENTORY	24.62	0.434	4.0	3.7	1	23.75	25.50
STOCKOUT QUANTITY	0.02	0.0078	0.11	0.11	0	0.005	0.035
STOCKOUT FREQUENCY	0.005	0.0012	0.016	0.016	0	0.003	0.007
REPLENISHMENT QUANTITY	4.04	0.066	0.86	0.79	2	3.91	4.18
REPLENISHMENT FREQUENCY	0.158	0.0017	0.024	0.024	6	0.154	0.161
COST PER PERIOD	36.68	0.817	11.6	11.6	6	35.07	38.30

Figure 1b

ACTUAL VALUES LESS FORECASTS:							
(1) 1-PERIOD FORECASTS:							
PERIOD-END INVENTORY	0.09	0.406	5.7	5.7	0	-0.70	0.89
STOCKOUT QUANTITY	0.07	0.0224	0.32	0.32	0	0.022	0.110
STOCKOUT FREQUENCY	0.012	0.0037	0.052	0.052	0	0.005	0.019
REPLENISHMENT QUANTITY	-0.07	0.109	1.55	1.55	0	-0.28	0.15
REPLENISHMENT FREQUENCY	-0.000	0.0044	0.063	0.063	0	-0.009	0.008
COST PER PERIOD	6.62	2.161	30.6	30.6	0	2.36	10.98
(2) 2-PERIOD FORECASTS:							
PERIOD-END INVENTORY	0.06	0.247	5.0	4.7	0	-0.43	0.55
STOCKOUT QUANTITY	0.06	0.0232	0.33	0.33	0	0.015	0.106
STOCKOUT FREQUENCY	0.012	0.0036	0.051	0.051	0	0.005	0.019
REPLENISHMENT QUANTITY	-0.06	0.057	1.25	1.18	0	-0.17	0.06
REPLENISHMENT FREQUENCY	-0.002	0.0035	0.050	0.050	0	-0.009	0.005
COST PER PERIOD	5.92	2.222	31.4	31.4	0	1.54	10.31

Figure 1c

10.

Reproduced from best available copy.

STATISTICAL PROPERTIES OF INVENTORY SYSTEMS

EXPERIMENT # : 5454
TIME/DATE : 1737/072073

SYSTEM ASSUMPTIONS:

- QUANTITY DEMANDED HAS NEGATIVE BINOMIAL DISTRIBUTION, MEAN = 4, VARIANCE/MEAN RATIO = 3
- STOCKOUTS BACKLOGGED, REPLENISHMENT LEAD TIME = 4
- S,S REPLENISHMENT POLICY
- COST RATIOS: C(OUT)/C(IN) = 99, C(FIX)/C(IN) = 64

SAMPLE STATISTICS OF SYSTEM OPERATING CHARACTERISTICS, SAMPLE SIZE 200 CYCLES OF 26 PERIODS:

OPERATING CHARACTERISTIC	SAMPLE MEAN OF O.C.	S.D. OF MEAN	SAMPLE S.D. OF O.C.	RESIDUAL S.D.	AUTOREGRESSIVE ORDER	CONFIDENCE POINTS 0.025L	0.975U
•DECISION RULE TO SET (S,S): DYNAMIC PROGRAM (OPTIMAL) (S,S) = (57,33), EXPECTED COST = 40.50							
MEAN OVER 26 PERIODS OF:							
PERIOD-END INVENTORY	26.73	0.237	3.4	3.4	0	26.27	27.20
STOCKOUT QUANTITY	0.03	0.0074	0.11	0.11	0	0.014	0.043
STOCKOUT FREQUENCY	0.007	0.0013	0.019	0.019	0	0.004	0.010
REPLENISHMENT QUANTITY	4.02	0.053	0.76	0.76	0	3.91	4.12
REPLENISHMENT FREQUENCY	0.149	0.0020	0.028	0.028	0	0.145	0.153
COST PER PERIOD	39.07	0.697	9.9	9.9	0	37.70	40.45
•DECISION RULE TO SET (S,S): MORSE UNIT NORMAL APPROXIMATION (S,S) = (55,32), EXPECTED COST = 40.63							
MEAN OVER 26 PERIODS OF:							
PERIOD-END INVENTORY	25.15	0.239	3.4	3.4	0	24.68	25.62
STOCKOUT QUANTITY	0.05	0.0106	0.15	0.15	0	0.025	0.066
STOCKOUT FREQUENCY	0.011	0.0020	0.028	0.028	0	0.007	0.015
REPLENISHMENT QUANTITY	4.02	0.051	0.72	0.72	0	3.92	4.12
REPLENISHMENT FREQUENCY	0.155	0.0019	0.027	0.027	0	0.151	0.159
COST PER PERIOD	39.58	1.011	14.3	14.3	0	37.59	41.57
•DECISION RULE TO SET (S,S): STATISTICAL NORMAL APPROXIMATION (PARAMETERS REVISED EVERY 26 PERIODS USING 26 PERIODS DATA)							
BIG S	53.9	0.50	7.0	7.0	0	53.0	54.9
LITTLE S	31.4	0.38	5.4	5.4	0	30.6	32.1
MEAN OVER 26 PERIODS OF:							
PERIOD-END INVENTORY	24.81	0.339	6.8	6.4	1	24.14	25.48
STOCKOUT QUANTITY	0.11	0.0220	0.31	0.31	0	0.068	0.155
STOCKOUT FREQUENCY	0.021	0.0032	0.045	0.045	0	0.014	0.027
REPLENISHMENT QUANTITY	4.02	0.058	0.81	0.81	0	3.90	4.13
REPLENISHMENT FREQUENCY	0.156	0.0020	0.028	0.028	0	0.152	0.160
COST PER PERIOD	45.82	2.045	28.9	28.9	0	41.79	49.86
FORECASTS:							
FROM 26 PERIODS' HISTORY:							
PERIOD-END INVENTORY	24.51	0.274	3.9	3.9	0	23.97	25.05
STOCKOUT QUANTITY	0.02	0.0070	0.10	0.10	0	0.009	0.036
STOCKOUT FREQUENCY	0.007	0.0012	0.017	0.017	0	0.004	0.009
REPLENISHMENT QUANTITY	4.07	0.066	0.94	0.94	0	3.94	4.20
REPLENISHMENT FREQUENCY	0.157	0.0019	0.027	0.027	0	0.153	0.161
COST PER PERIOD	36.78	0.761	10.8	10.8	0	35.28	38.28
ACTUAL VALUES LESS FORECASTS: (26-PERIOD FORECASTS):							
PERIOD-END INVENTORY	0.30	0.154	4.4	3.9	2	-0.01	0.60
STOCKOUT QUANTITY	0.09	0.0234	0.33	0.33	0	0.043	0.135
STOCKOUT FREQUENCY	0.014	0.0034	0.048	0.048	0	0.007	0.021
REPLENISHMENT QUANTITY	-0.05	0.036	1.00	0.89	2	-0.12	0.02
REPLENISHMENT FREQUENCY	-0.001	0.0019	0.037	0.035	1	-0.004	0.003
COST PER PERIOD	9.04	2.279	32.2	32.2	0	4.55	13.54

Figure 12

TABLE A1 AVERAGE TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND
REPLENISHMENT LEADTIME = 4

(X,Y) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION)

VALUES FOR RULES OTHER THAN THE OPTIMAL OP ARE % EXCESS OVER OP VALUE

MEAN	C(OUT) /C(IN)	C(FIX) /C(IN)	OP	UN	(13,13)	(13,26)	(26,26)
2	4	8	10.7	3.2	24.7	10.0	14.2
4	4	8	14.9	5.2	17.7	15.7	8.1
2	9	8	13.6	2.1	14.8	12.4	6.2
4	9	8	18.7	1.8	21.6	17.3	15.2
2	99	8	22.1	5.3	43.0	28.9	26.8
4	99	8	29.4	1.4	67.5	16.2	27.6
2	4	32	14.3	3.4	11.9	3.2	7.3
4	4	32	20.1	2.6	7.7	2.0	5.3
8	4	32	28.3	2.1	11.4	4.8	5.9
16	4	32	39.7	1.9	9.3	5.6	6.4
24	4	32	48.4	1.9	10.0	5.7	4.9
2	9	32	17.3	2.1	16.6	5.8	7.9
4	9	32	24.1	1.3	11.6	13.5	7.5
8	9	32	33.7	0.7	22.2	3.7	5.0
16	9	32	47.1	0.6	10.1	3.3	4.3
24	9	32	57.2	0.9	10.3	4.0	5.8
2	99	32	26.0	2.7	65.5	30.9	30.2
4	99	32	35.2	0.8	44.5	25.5	13.2
8	99	32	48.0	0.3	20.4	9.2	15.4
16	99	32	66.1	0.0	20.5	11.7	16.0
24	99	32	79.9	0.2	25.5	17.3	12.4
2	4	64	17.7	1.9	6.5	5.2	4.3
4	4	64	25.0	1.2	7.8	5.7	5.0
8	4	64	35.2	1.1	5.5	5.8	2.1
16	4	64	49.6	1.1	7.9	5.8	2.5
24	4	64	60.5	1.0	7.8	5.1	1.3
36	9	64	78.3	1.6	2.4	1.8	5.5
2	9	64	20.8	0.9	13.5	11.9	8.9
4	9	64	29.2	0.9	10.2	5.3	1.1
8	9	64	40.9	0.8	13.2	8.7	7.2
16	9	64	57.4	0.6	5.9	7.8	3.6
24	9	64	70.0	0.7	11.7	3.3	4.4
2	99	64	29.7	1.9	30.7	2.4	19.5
4	99	64	40.5	0.3	27.2	5.2	13.2
8	99	64	55.7	0.4	31.8	17.1	10.1
16	99	64	77.2	0.1	12.4	9.8	7.5
24	99	64	93.7	0.1	25.2	-0.7	18.3

Figure 2

12

TABLE A7 ESTIMATED BIAS OF FORECAST OF TOTAL COST

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND
REPLENISHMENT LEADTIME = 4

(X,Y,Z) = (REVISION INTERVAL, NO. OF PERIODS DEMAND DATA USED FOR REVISION, NO. OF PERIODS DEMAND DATA USED TO FORECAST)
COLUMNS (2) - (5) % EXCESS OF MEAN ACTUAL COST OVER MEAN FORECAST COST
COLUMNS (6) - (9) + : BIAS FOR O.C.: POSITIVE ; - : NEGATIVE ; # : SIGNIFICANTLY POSITIVE ; = : SIGNIFICANTLY NEGATIVE
SUBCOLUMNS: PERIOD-END INVENTORY, STOCKOUT QUANTITY, STOCKOUT FREQUENCY, REPLENISHMENT QUANTITY, REPLENISHMENT FREQUENCY, COST

(1) MEAN	C(OUT)	C(FIX)	(2) (13,13,13)	(3) (13,26,13)	(4) (13,26,26)	(5) (26,26,26)	(6) (13,13,13)	(7) (13,26,13)	(8) (13,26,26)	(9) (26,26,26)
2	4	8	25.5	15.9	15.3	15.5	# # # - - #	# # # - - #	# # # - - #	# # # - + #
4	4	8	23.0	16.3	14.2	13.8	# # # - - #	# # # - + #	# # # - - #	# # # + - #
2	9	8	29.5	22.7	22.3	17.8	# # # - + #	# # # - - #	# # # - - #	# # # + - #
4	9	8	28.3	21.8	20.8	21.8	# # # - + #	# # # - - #	# # # - - #	# # # - - #
2	99	8	47.6	41.0	38.7	38.5	# # # - - #	+ # # - - #	# # # - - #	+ # # - + #
4	99	8	50.4	30.0	29.3	35.5	# # # + + #	+ # # + + #	+ # # - + #	# # # - + #
2	4	32	16.4	9.6	7.9	11.4	+ # # + + #	# # # - + #	# # # - - #	# # # + + #
4	4	32	15.9	9.7	8.8	9.4	# # # - - #	+ # # - + #	+ # # - + #	# # # + + #
8	4	32	15.1	8.5	8.2	8.0	# # # - + #	# # # - + #	+ # # - - #	# # # - - #
16	4	32	15.0	8.8	8.5	9.0	# # # + + #	# # # - + #	# # + - + #	# # # - - #
24	4	32	13.9	9.0	8.1	8.5	# # # - - #	# # # + - #	# # # - - #	# # # - + #
2	9	32	22.4	14.8	13.2	13.8	# # # - - #	+ # # + + #	+ # # + - #	+ # # - + #
4	9	32	19.5	17.1	15.1	11.9	# # # + + #	+ # # + + #	+ # # + - #	+ # # + + #
8	9	32	25.3	11.4	9.8	10.6	# # # + + #	+ # # - + #	# # # - + #	# # # + + #
16	9	32	16.9	8.6	8.2	9.1	+ # # - - #	+ # # + - #	+ # # + - #	# # # + + #
24	9	32	14.4	6.6	6.4	8.8	# # # - - #	+ # + + + #	+ # # - + #	# # # - - #
2	99	32	50.1	33.5	35.2	36.6	+ # # + # #	- # # - - #	# # # - - #	+ # # + # #
4	99	32	41.4	30.2	29.3	23.0	# # # + + #	- # # - - #	+ # # - - #	+ # # + - #
8	99	32	27.4	15.9	16.1	22.6	+ # # + + #	- # # - - #	+ # # - - #	+ # # + + #
16	99	32	24.7	18.9	17.6	21.6	+ # # - - #	- # # + = #	+ # # - - #	+ # # + + #
24	99	32	24.6	20.5	18.8	16.1	+ # # - + #	+ # # - - #	+ # # - - #	+ # # - + #
2	4	64	13.4	9.0	7.8	9.7	# # # - + #	+ # # + + #	+ # + + + #	# # + + + #
4	4	64	13.2	8.1	6.5	7.9	# # # + + #	# # + + + #	# # + + + #	# # # + + #
8	4	64	11.0	7.0	7.1	5.5	+ # # + + #	# # + - + #	# # + - - #	# # + - - #
16	4	64	11.8	6.7	7.0	6.3	# # # + + #	+ # # - - #	# # # - - #	# # # + + #
24	4	64	11.3	6.3	5.8	4.3	# # # - - #	+ # + - - #	+ # + - - #	+ # # - - #
36	4	64	2.7	2.4	2.2	6.8	+ + + - + #	+ # # - + #	+ # + - - #	+ # # - - #
2	9	64	17.2	16.5	14.1	12.2	+ # # - + #	+ # # + + #	+ # # - + #	+ # # - + #
4	9	64	15.0	8.9	7.5	7.1	+ # # - - #	+ # # + + #	+ # # - - #	+ # # + + #
8	9	64	16.9	11.8	10.4	9.0	# # # + + #	# # # + + #	+ # # - + #	+ # # - - #
16	9	64	12.1	9.2	9.0	7.5	# # # + + #	+ # # + + #	+ # # + + #	+ # # + + #
24	9	64	14.2	7.1	6.3	7.4	# # # + + #	+ # # - - #	+ # # - - #	+ # # + - #
2	99	64	35.3	14.9	14.7	27.7	- # # - + #	- # # + + #	+ # # + + #	+ # # - + #
4	99	64	32.7	15.5	13.9	19.7	+ # # + + #	+ # # - - #	+ # # - - #	+ # # - - #
8	99	64	31.8	23.3	19.8	15.3	- # # - + #	+ # # - - #	+ # # - - #	+ # # + + #
16	99	64	18.5	14.2	14.2	12.3	- # # - + #	+ # # + + #	+ # # - + #	+ # # - + #
24	99	64	26.2	6.4	6.2	18.1	+ # # + + #	+ # # - - #	+ # # - + #	+ # # - + #

Figure 3

12.a

TABLE A9 VALUES FOR (S,S)

NEGATIVE BINOMIAL DISTRIBUTION FOR DEMAND
REPLENISHMENT LEADTIME = 4

A = (13,13), B = (13,26), C = (26,26), WHERE
(X,Y) : X = REVISION INTERVAL ; Y = # PERIODS DATA USED TO REVISE PARAMETERS

MEAN	C(OUT)	C(FIX)	BIG S					LITTLE S					BIG S - LITTLE S					EOQ
			DP	UN	A	B	C	DP	UN	A	B	C	DP	UN	A	B	C	
2	4	8	18	18	18.0	17.0	17.6	10	12	12.3	11.5	11.9	8	6	5.8	5.6	5.7	5.7
4	4	9	31	32	30.3	31.2	30.1	20	24	22.4	23.2	22.1	11	8	7.9	8.0	8.0	8.0
2	9	8	21	21	19.3	19.7	19.2	13	15	13.8	14.2	13.7	8	6	5.6	5.6	5.5	5.7
4	9	8	35	35	34.1	34.1	33.9	24	27	26.2	26.2	26.0	11	8	7.9	7.9	7.9	8.0
2	99	8	29	26	25.0	25.4	25.8	22	20	19.5	20.0	20.2	7	6	5.6	5.6	5.6	5.7
4	99	9	46	43	43.3	42.5	42.4	36	35	35.3	34.6	34.4	10	8	8.0	7.9	8.0	8.0
2	4	32	22	21	19.9	19.7	19.7	7	10	8.8	8.6	8.5	15	11	11.1	11.2	11.2	11.3
4	4	32	36	36	33.3	33.6	34.2	17	20	17.7	17.8	18.3	19	16	15.6	15.8	16.0	16.0
8	4	32	63	63	60.8	60.3	60.1	36	40	38.1	37.7	37.5	27	23	22.7	22.6	22.6	22.6
16	4	32	110	112	106.8	108.2	109.8	74	80	75.1	76.3	76.8	36	32	31.7	31.9	32.0	32.0
24	4	32	155	159	154.1	155.0	154.6	113	120	115.7	115.9	115.6	42	39	38.5	39.0	39.0	39.2
2	9	32	25	24	23.6	23.4	23.5	11	13	12.4	12.1	12.2	14	11	11.3	11.3	11.3	11.3
4	9	32	40	40	38.6	38.8	39.1	22	24	22.8	22.9	23.1	18	16	15.8	15.9	15.9	16.0
8	9	32	69	68	67.6	65.9	66.9	43	45	44.9	43.6	44.3	25	23	22.8	22.3	22.6	22.6
16	9	32	117	119	116.6	117.3	118.3	84	87	84.8	85.5	86.3	33	32	31.8	31.8	32.0	32.0
24	9	32	164	168	166.1	168.2	166.6	125	129	128.0	128.8	127.5	39	39	35.2	39.4	39.1	39.2
2	99	32	33	30	28.6	30.5	29.5	20	19	17.7	19.1	18.3	13	11	10.9	11.4	11.2	11.3
4	99	32	52	49	48.9	49.6	48.7	34	33	32.9	33.5	32.7	18	16	16.1	16.1	16.0	16.0
8	99	32	83	81	80.6	81.7	80.0	59	58	57.9	58.9	57.4	24	23	22.6	22.8	22.5	22.6
16	99	32	137	138	135.9	135.8	136.9	106	106	104.0	103.9	104.9	31	32	31.9	31.9	32.0	32.0
24	99	32	187	190	187.6	190.0	189.3	152	151	149.3	150.9	150.3	35	39	38.3	39.1	39.0	39.2
2	4	64	25	24	22.1	22.5	22.5	6	8	6.6	6.8	6.8	19	16	15.4	15.7	15.7	16.0
4	4	64	41	40	38.4	39.2	38.7	15	17	15.9	16.3	16.0	26	23	22.6	22.9	22.7	22.6
8	4	64	70	68	65.3	66.8	65.8	33	36	33.5	34.6	33.9	37	32	31.7	32.2	32.0	32.0
16	4	64	120	120	116.6	115.9	116.1	70	75	71.5	70.9	71.0	50	45	45.1	44.9	45.1	45.3
24	4	64	167	169	164.6	165.0	163.8	108	114	109.3	109.6	108.6	59	55	55.3	55.4	55.2	55.4
94	9	64	164	177	162.5	161.6	174.6	106	109	94.3	93.6	106.8	58	68	68.2	[illegible]	67.8	67.9
2	9	64	28	27	27.3	26.7	26.9	10	11	11.3	10.9	10.9	18	16	16.0	15.9	16.0	16.0
4	9	64	46	45	44.0	44.0	43.1	20	22	21.3	21.3	20.7	26	23	22.7	22.6	22.4	22.6
8	9	64	75	75	74.2	74.5	74.4	40	43	42.1	42.4	42.2	35	32	32.0	32.1	32.1	32.0
16	9	64	128	129	127.0	129.1	127.7	81	84	81.8	83.6	82.5	47	45	45.2	45.5	45.2	45.3
24	9	64	177	180	178.2	179.2	179.7	121	125	122.8	123.7	124.0	56	55	55.4	55.5	55.6	55.4
2	99	64	37	34	32.6	33.9	33.8	19	18	17.2	17.9	17.9	18	16	15.5	16.0	15.9	16.0
4	99	64	57	55	52.8	53.7	53.9	33	32	30.5	31.2	31.4	24	23	22.2	22.5	22.6	22.6
8	99	64	90	88	88.7	87.0	88.4	57	56	56.6	55.3	56.5	33	32	32.1	31.7	31.9	32.0
16	99	64	148	148	148.4	149.6	148.6	104	103	103.2	104.0	103.3	44	45	45.2	45.6	45.3	45.3
24	99	64	200	204	291.4	203.6	205.1	149	149	146.4	148.0	149.5	51	55	55.1	55.5	55.6	55.4

Figure 4

SYSTEM OF 72 ITEMS WITH POISSON DEMAND DISTRIBUTIONS

STATISTICAL INFORMATION FROM 26-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 26 PERIODS

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM = 2037.8

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8.9	16
		28.6	31.9	39.5	43.4	56.6	30.4	33.5	36.2	13.8	19.3	28.8	38.1
MEAN													
2		3.9	4.4	5.5	6.0	7.8	4.2	4.6	5.0				
4		5.5	6.2	7.6	8.3	10.9	5.8	6.4	7.0				
8.9		8.2	9.1	11.5	12.6	16.3	8.7	9.6	10.5				
16		10.9	12.2	15.0	16.5	21.6	11.7	12.8	13.7				
LEADTIME													
0		9.0	9.8	11.5	12.9	17.5							
2		9.5	10.7	13.3	14.6	18.9							
4		10.0	11.4	14.7	16.0	20.2							
C(FIX)/C(IN)													
32		12.2	13.7	17.6									
64		16.4	18.2	22.0									
LEADTIME	C(FIX)/C(IN)												
0	32	3.8	4.2	5.0									
0	64	5.3	5.7	6.6									
2	32	4.1	4.6	5.9									
2	64	5.5	6.0	7.4									
4	32	4.3	5.0	6.7									
4	64	5.7	6.4	8.0									

Figure 5a

SYSTEM OF 72 ITEMS WITH POISSON DEMAND DISTRIBUTIONS

STATISTICAL INFORMATION FROM 26-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 26 PERIODS

SOURCES OF TOTAL COST
(% EXCESS OVER DP)

OVERALL AGGREGATE FOR SYSTEM = 2.6

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8.9	16
		1.2	2.0	4.1	3.6	1.9	1.3	2.4	3.9	3.1	2.2	2.6	2.6
MEAN													
	2	1.7	2.3	4.8	3.8	2.6	1.1	1.8	6.1				
	4	1.3	2.2	2.8	2.6	1.9	0.5	2.2	3.7				
	8.9	1.1	1.5	4.6	3.7	1.7	1.0	2.4	4.1				
	15	1.1	2.3	4.0	3.9	1.7	1.9	2.8	3.0				
LEADTIME													
	0	0.7	1.1	1.9	1.9	0.8							
	2	1.0	2.0	3.9	3.4	1.7							
	4	1.9	2.9	6.0	5.2	2.9							
C(FIX)/C(IN)													
	32	1.6	2.8	5.6									
	64	0.9	1.5	2.9									
LEADTIME	C(FIX)/C(IN)												
0	32	1.1	1.8	2.6									
0	64	0.4	0.6	1.4									
2	32	1.4	2.8	5.2									
2	64	0.6	1.4	2.9									
4	32	2.2	3.5	8.5									
4	64	1.7	2.4	4.1									

Figure 5b

SYSTEM OF 72 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS

STATISTICAL INFORMATION FROM 26-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 26 PERIODS

SOURCES OF TOTAL COST

OVERALL AGGREGATE FOR SYSTEM = 2480.3

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8.9	16
		26.0	30.3	43.7	45.1	54.9	27.7	33.8	38.6	14.9	19.7	27.4	38.0
MEAN													
2		3.6	4.3	7.0	6.8	8.1	4.0	5.1	5.8				
4		5.1	5.9	8.6	8.9	10.8	5.5	6.7	7.6				
8.9		7.1	8.4	11.9	12.3	15.1	7.6	9.2	10.6				
16		10.1	11.6	16.2	17.1	20.9	10.6	12.8	14.6				
LEADTIME													
0		7.7	8.6	11.4	12.0	15.6							
2		8.7	10.2	14.9	15.2	18.6							
4		9.7	11.5	17.4	17.9	20.7							
C(FIX)/C(IN)													
32		11.5	13.5	20.2									
64		14.6	16.8	23.5									
LEADTIME	C(FIX)/C(IN)												
0	32	3.3	3.7	5.1									
0	64	4.4	4.9	6.3									
2	32	3.8	4.5	6.8									
2	64	4.9	5.7	8.0									
4	32	4.4	5.2	8.3									
4	64	5.3	6.3	9.1									

Figure 5c

16

SYSTEM OF 72 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS

STATISTICAL INFORMATION FROM 26-PERIOD DEMAND HISTORY USED FOR CONTROLS, REVISED EVERY 26 PERIODS

SOURCES OF TOTAL COST
(% EXCESS OVER OP)

OVERALL AGGREGATE FOR SYSTEM = 5.8

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8.9	16
		2.7	3.2	9.6	7.1	4.8	1.6	5.8	9.0	10.5	6.1	4.9	4.5
MEAN													
2		4.4	5.2	17.8	12.8	8.6	3.9	11.9	14.3				
4		3.4	2.9	10.2	7.1	5.3	2.2	5.6	9.6				
8.9		2.0	3.5	7.8	5.6	4.4	1.0	5.3	7.6				
15		2.3	2.5	7.5	6.0	3.3	0.9	4.1	7.6				
LEADTIME													
0		0.8	0.9	2.8	1.9	1.4							
2		2.2	3.2	10.0	6.7	5.1							
4		4.7	5.2	14.2	11.2	7.1							
C(FIX)/C(IN)													
32		3.5	4.0	11.5									
64		2.1	2.7	8.1									
LEADTIME	C(FIX)/C(IN)												
0	32	1.2	1.5	2.6									
0	64	0.5	0.4	2.9									
2	32	2.5	3.3	11.8									
2	64	2.0	3.0	8.6									
4	32	6.3	6.4	17.4									
4	64	3.5	4.2	11.5									

Figure 5d

SYSTEM OF 72 ITEMS WITH NEGATIVE BINOMIAL DEMAND DISTRIBUTIONS

FORECASTS, MADE EVERY 26 PERIODS, USING A 26-PERIOD HISTORY; CONTROL REVISION HISTORY 26 PERIODS

SOURCES OF TOTAL COST
(% UNDERESTIMATE OF ACTUAL)

OVERALL AGGREGATE FOR SYSTEM = 9.3

		C(OUT)/C(IN)			C(FIX)/C(IN)		LEADTIME			MEAN			
		4	9	99	32	64	0	2	4	2	4	8.9	16
		4.4	5.8	14.8	10.9	9.1	2.8	9.3	14.1	14.9	9.3	8.5	7.7
MEAN													
2		5.8	8.7	23.6	16.8	13.3	4.6	15.5	21.5				
4		4.5	5.6	14.8	10.7	8.2	2.7	8.8	14.6				
8.9		3.7	5.4	13.8	9.9	7.6	2.7	8.5	13.0				
16		4.3	5.0	11.7	9.2	6.4	2.1	7.6	11.8				
LEADTIME													
0		0.9	1.3	5.1	2.6	2.9							
2		3.5	5.3	15.3	10.4	8.3							
4		7.9	9.5	20.7	16.8	11.9							
C(FIX)/C(IN)													
32		5.2	6.5	17.0									
64		3.8	5.2	12.9									
LEADTIME	C(FIX)/C(IN)												
0	32	0.7	1.4	4.7									
0	64	1.0	1.2	5.5									
2	32	4.5	5.9	16.8									
2	64	2.8	4.9	14.1									
4	32	9.1	10.8	24.6									
4	64	6.9	8.5	17.0									

Figure 6